Solution Writing Guide

1) Have a Plan
   a) Express ideas clearly and concisely
   b) Organization and ordering is important
   c) Define items at the beginning of the proof
   d) Create a simple outline of your solution

Sample Outline:

Stuff to define: ABCD, h_a, S, [A], AXYZ.
Order of things to prove:

1. Volume ABCD = rS/3 (lemma)
2. Show altitude AXYZ = $h_a - 2r$
3. Use similarity to get $a = r - 2r^2/h_a$
4. equate volumes to get $1/h_a = A/(rS)$,
5. sub 4 into 3 and add

2) Readers Are Not Interpreters
   a) Use blank paper
   b) Respect margins
   c) Write horizontally
   d) Suggested header: 'Page _ of _'
   e) Print clearly: no cursive
   f) Use pen
   g) Made a mistake? Draw a single horizontal line to cross out
   h) Use addendums at the end "(*) Addendum" to include omitted material

3) Use Space
   a) Give each important definition or equation its own line
   b) When using algebra, avoid paragraphs. Use lines!
   c) Label all equations, lemmas, and cases clearly
   d) Don't try to squish your writing to save paper

4) Think Backwards, Write Forwards

a) Treat your solutions as cookbook recipes: list the ingredients and explain when to add them to the pot

b) A clarifying explanation is better than none

5) Name Your Characters

a) Use definitions to help keep your writing clear, concise, and easy to follow

Sample Definitions:

Call the 100 integers $n_1, n_2, \ldots, n_{100}$.

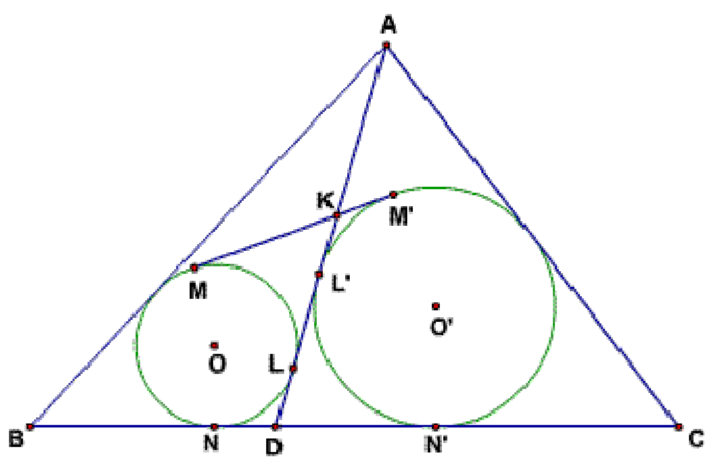
Let $S_k = n_1 + n_2 + \ldots + n_k$ for $k = 1, 2, \ldots, 100$.

6) A Picture Is Worth a Thousand Words

a) Diagrams save your reader work, and can help you trim extra wording

b) Even if the diagram is given in the problem, include it in your solution

Sample Diagram:



(Diagrams can save you and the reader a lot of work)

7) Solution Readers, not Mind readers

a) Justify every notable step of your solution
b) Cite theorems that have names (ex. Pythagorean Theorem), and you won't have to prove them
c) If you are unsure of the theorem's name, but you are sure that the theorem is well known, use the phrase "by a well known theorem…"
d) When using algebra, don't skip steps
e) Invoke symmetry or analogy when cases are precisely the same
f) Better to prove too much than too little

8) Follow the Lemmas

a) Lemmas are preliminary items that need to be proved before you can present your main proof
b) Dividing your proofs into lemmas simplify and organize otherwise complicated proofs

Sample Lemmas:

Lemma 1: JKBA is a rhombus.
Proof: …

---------------end lemma---------------

Lemma 2: MN || BC and MN is equidistant from lines AJ and BC.
Proof: …

9) Clear Casework

a) Some proofs require cases, which are specific instances when a variable is given a constant value or a set of values.
b) The whole solution will then be proved by finding the solution to each one of the cases

Sample Cases:

We divide our investigation into cases based on the smallest digit of each number.

Case 1: The smallest digit is 0. *(then….)*

Case 2: The smallest digit is 1. *(then….)*

Case 3: The smallest digit is 2. *(then….)*

10) <u>Proofread</u>
 a) Required of any well-written proof

11) <u>Bookends</u>
 a) Bookends divide sections of the proof
 b) Sample bookends include “---end lemma---”, “QED”, or some symbol

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Sample Proof

Problem: Let p(x) be a polynomial with degree 98 such that p(n) = 1/n for n= 1, 2, 3, 4, . . . , 99. Determine p(100).

Solution: Let

$$r(x) = x\,(p(x) - 1/x) = x\,p(x) - 1. \qquad (1)$$

Since p(x) is a polynomial with degree 98, r(x) is a polynomial with degree 99. Since r(x) = x (p(x) - 1/x), and we are given that (p(x) - 1/x) = 0 for x = 1, 2, 3, . . . , 99,

r(x) has roots 1, 2, . . . , 99.

Since r(x) has degree 99, these are the only roots of r(x), which must thus have the form

$$r(x) = c(x - 1)(x - 2)(x - 3) \ldots (x - 99) \qquad (2)$$

for some constant c. To find c, we first let x = 0 in equation (1), yielding r(0) = -1. Letting x = 0 in (2) yields r(0) = -c(99!); hence, c = 1/99!. Thus, we have

$$r(x) = (x - 1)(x - 2)(x - 3) \ldots (x - 99)/99! \qquad (3)$$

We can combine equations (1) and (3) and let x = 100 to find

$$100p(100) - 1 = (100 - 1)(100 - 2)(100 - 3) \ldots (100 - 99)/99!$$
$$100p(100) - 1 = 99!/99! = 1$$
$$p(100) = 1/50.$$