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The Simple Joy of Mathematics

Many people including myself grew up believing that mathematics was a tool. In elementary and middle school, the fundamentals of mathematics are not designed to inspire or to enlighten, but rather to learn how to get by in life. Always, textbooks would refer to oddball questions of "Which person ate more candy if Albert ate twice as much of 4 less candies than Bob?" trying to make math more "fun" for the students. People would assume that mathematics itself is so boring that a student could not sit through lectures; illustrations, examples, and practical applications were inserted to keep hold of our attentions.

As I have matured and grown, I understand that complex mathematics can lead to great improvements in the practical lives of common people, such as using Fourier transforms to better model voice recognition technology or using complex data analysis to track all of the polygons in an intense simulation game. However, the better lesson that I have learned is that modeling the real world is not the fundamental goal of mathematics. Rather, it is the simple elegance that math as play can bring to the mind.

In *Lockhart's Lament*, a brilliant piece penned by Lockhart published in the American Mathematical Society Journal, Lockhart comments on how the true elegance of mathematics is being buried under a pile of required tests, curriculums and other mandatory materials[1]. He encourages a return to the more fundamental games in math, the discovery of patterns and a greater understanding of the fascinating history that connects the thread of mathematics. But while math puzzles such as the Knight's Tour or the Bridges of Königsberg are fascinating, I

[1] Lockhart's Lament

would like to direct your attention to another puzzle that has captivated my attention: The Brachistochrone Problem.

In 1697, the great mathematician Johann Bernoulli issued a challenge to the greatest thinkers of Europe: How would one discover the path that a threaded bead could take which, if acted upon only by gravity, would travel in the least time? While the word "least" immediately tips the reader off that the solution would involve minimizing some function, the difficulty in this puzzle is that a numerical solution is not what was being looked for. Rather, a family of functions must be derived in order for completion. Standard calculus methods were insufficient, but out of this puzzle came some great realizations in geometry and for future branches of mathematics.

When Bernoulli posed this question, he had already solved one form of it. Instead of using a bead on a string, he imagined a ray of light traveling through the medium. This light wave is known to always take the path of least time between two points. Therefore if we assume that the light wave is being affected by gravity, we can apply Snell's law infinitely many number of times to derive a general equation[2].

While I very much respect Bernoulli's take on the problem, it is Newton that I was utterly fascinated by. Upon receiving at his home at 4 pm on January 29, 1697, he immediately set off to work on this challenge. By morning of the next day, he had invented the calculus of variations and applied it to solve this problem[3].

In fact, the solution to this problem is far from trivial. While Bernoulli's ideas were brilliant in their own respect, a key part of puzzles is how they can lead to greater leaps in

[2] *When Least is Best*, p. 212
[3] *Cosmos*, p74

understanding. In solving this problem, Newton opened up an entire field of analytics. The first step is to understand that in this circumstance, what must be minimized is not a value, but rather, a family of functions. Instead, what must be minimized in the equation:

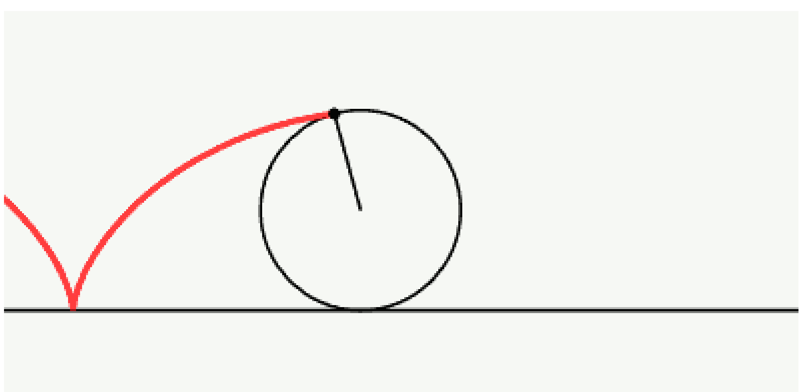
$$T_B = \int_0^L \sqrt{\frac{1+(y')^2}{2gy}}\,dx$$

Is the function y, where it is unknown what is needed. The full derivation is rather complicated, and deals with minimizing the work function between the true path and variations along this path. Through taking several derivatives with respect to multiple variables, the final solution can be found:

$$x = \alpha[\beta - \sin\beta]$$

$$y = \alpha[1 - \cos\beta]$$

Which is quite surprising: This function is neither an odd logarithm nor a perfect circle: It is the equation of the cycloid. This shape is the interesting result if you place a pen on a circle, and allow it to rotate. What connections there are in the game that we are playing!



The Path of the Cycloid[4]

Another interesting note is how this curve was soon discovered to be the solution to an equally famous problem, the tautochrone curve. This means that not only is the cycloid the path

[4] Wikimedia

of least time, but that regardless of where the item starts on the curve, each of the balls will arrive at the final destination at the exact same time!

However, no matter how much we might improve on the ideas and the mathematics behind these families of curves, this system is inherently impossible. All of the equations used rely on a system where no friction exists and a perfect curve is possible. Therefore, however we create the equation, there are imprecisions when the “problem” is in the real world.

But let us think back to the original purpose of math. Why did we create this problem in the first place? There is not an overwhelming demand for mathematicians to create these curves, nor is there a practical application for the curve of least time. Rather, all of these things exist as toys for the mind. They are interesting ideas and puzzles to be analyzed! For a mathematician, all of mathematics is fun and games; the goal is not to solve a problem, but satisfy one’s curiosity.

I encourage my peers to go out and look for similar discoveries, because the knowledge gained from working through such puzzles can prove to be vastly beneficial for the future. I see math as enriching to my life, something that I will carry with me no matter where I go.

Word Count: 994

Works Cited

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