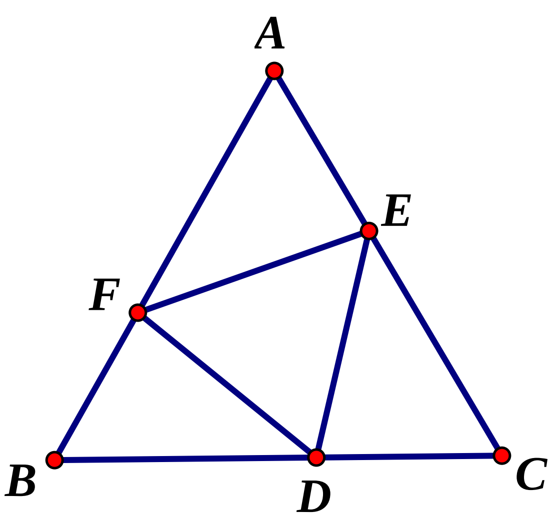
WSMA Math Bowl – March 2, 2013

Time Attack

1	Let *E* be the midpoint of side *AD* in rectangle *ABCD* and *F* be the point of intersection of *BE* and *AC*. Find *AF*:*FC*.
2	What is the probability of not rolling a sum of 7 when rolling a pair of standard dice?
3	If $x + y = 2$ and $xy = 1$, find the value of $x^{100} + y^{100}$.
4	How many subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are there?
5	a_1=1! a_2=1!+2! a_3=1!+2!+3! a_4=1!+2!+3!+4! … a_{2013}=1!+2!+…2013! Find the number of square numbers from a_1 to a_{2013}.
6	The intersection of the diagonals of parallelogram *ABCD* is *E*. If angle *BEC*= angle *BAD*, find the value of $\frac{CD}{AC}$.
7	Austin and Evan are solving a math problem. The probabliities that they each get the right answer are $\frac{1}{4}$ and $\frac{1}{2}$ respectively. Find the probability that at least one of them gets it right.
8	Some amount of a 6% salt solution is mixed with 2ml of a 15% salt solution to obtain a 12% salt solution. How much of the 6% solution should be used?
9	How many different 6-letter patterns can be created from the letters in the word CROCODILE in which any occurrence of the letter O is immediately followed by the other O?
10	There are *n* papers stacked in a file for the Math Bowl. Andrew, who has nothing to do, divides the whole stack into groups of 3, then groups of 5, and finally groups of 7. However, there are 2, 3, and 2 papers left over respectively when he attempts these groupings. Find the 2^{nd} least possible value of *n*.
11	Let $T = \frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + \cdots + \frac{1}{2012*2013}$. Find the greatest integer that is less or equal to *T*.

12	Find the number of integer solutions that satisfy the equation $x^2 - 3y^2 = 17$.
13	The vertex of $f(x) = ax^2 + bx + c$ is at (2,3) and the difference between the x-coordinates of the x-intercepts of $f(x)$ is 2. Find the value of *a*+*b*+*c*.
14	Let $LCM(a, b)$ denote the least common multiple and $GCF(a, b)$ the greatest common factorof *a* and *b*. If *a* and *b* are positive integers that satisfy $GCF(a, b) = 500$ and $LCM(a, b) = 2000$, find the value of *ab*.
15	Varun is taking a true-false test with 17 questions, but he only knows the answer to the first question! How many ways are there for him to get exactly 13 of the problems correct?
16	Find $a + b + c + d$ where a, b, c, and d are digits in the expression below: $40! = 815915283247897734345a11b695961c589427d000000000$
17	Equilateral triangle *ABC* contains another equilateral triangle *DEF* so that point *F* is on side *AB*, point *D* is on side *BC*, and point *E* is on side *AC*. If side *AB* and side *DF* are perpendicular, find the ratio of the area of triangle *DEF* to the area of triangle *ABC*. 
18	A bag contains marbles that are red, green, or orange. The ratio of red:green:orange marbles is 5:6:7. If there are 49 orange marbles, find the difference between the number of green and red marbles.
19	In triangle *ABC*, angle *A*=60, angle *C*=45, and side *c*=10. Find the area of this triangle.
20	Find the range of *a* that satisfies the following inequality $a(x-1)^2 > 2x^2 - 2x - 2$
21	If *x* and *y* are positive integers and 2*x*+*y*=6, find the maximum possible value of x^2y.

22	The following fraction (in which A, B, C, and D represent digits and CBA and ADA are three digit numbers) was expressed as a decimal: $$\frac{CBA}{ADA} = 0.MATHMATHMATH\ldots$$ If A, B, C, D, M, T, and H are all integers between 0 and 9 inclusive and A, C, M, and T are not 0, find the sum of all possible values of $\frac{CBA}{ADA}$
23	Find the area of triangle ABC where $A(3, 5)$, $B(\sqrt{2}, 3)$, and $C(0, 0)$.
24	How many numbers formed by using the digits 1, 2, 3, 4, and 5 (exactly once each) are greater than 32000?