A *ratio* compares two amounts. You can use ratios to compare numbers in different ways. For instance, you can compare part of a group to the whole collection:

"Four out of five kids like math!"

Or, you can compare part of a group to another part:

"For every four kids that like math, one more kid doesn't."

Ratios

A *ratio* is the relationship between two numbers or values.

Example: There are 10 apples in a bag. 4 apples are yellow and 6 are red. What is the ratio of yellow apples to red?

Yellow apples = 4
Red apples = 6

The ratio of yellow apples to red is 4 to 6.
We write this as **4 : 6** or as the fraction **4 / 6**.
The numbers 4 and 6 are the terms of the ratio.

Example: What is the ratio of yellow apples to the total number of apples?

Total number of apples = 10.
Ratio of yellow to total = 4:10 or 4/10

Ratio 4/10 is the fractional portion of yellow apples to the total.

Similarly for the red apples, the ratio would be 6:10 or 6/10.

You always reduce ratios in the same way you reduce fractions.

Writing Ratios in Simplest Terms

When the numbers in a ratio are large, it can be difficult to see the relation between them. It often helps to reduce a ratio to its simplest terms.

To write a ratio in simplest terms, divide both parts by their **greatest common factor** (GCF).

Reducing a ratio to simplest terms does not change the relationship between the numbers. Suppose a ratio of chocolate donuts to maple bars is 6:4. Notice there are three chocolate donuts for every two maple bars. We can reduce the ratio 6:4 to 3:2.

Reducing a ratio is very much like reducing a fraction.

Common Mistakes With Ratios

➢*Warning:* A common mistake with ratios is in choosing the term for the 'total'. Read the problem carefully! Does it ask to compare a number to the total amount, or to the remaining amount?

Example: Suppose 20% of the kids in a class wore a blue shirt, and the rest wore red. What is the ratio of blue to red?

If you don't read carefully, you might convert 20% to a ratio and answer 2 : 10 or 1 : 5. This would be incorrect.
Since 20% wore blue and 80% wore red, the correct answer is 2 : 8, but you are in Math Club and will reduce your answer and say 1 : 4.

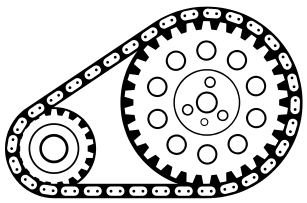
Example: What is the ratio of temperature in Fahrenheit to Celsius?

The temperatures of freezing and boiling water in Fahrenheit are 32 to 212. Freezing and boiling water in Celsius is 0 to 100.

To compare, notice a 100C change is equivalent to 212 – 32 = 180°F. Therefore the ratio is 180 : 100 which reduces to 9 : 5.

Gear Ratios

Gears and chains are another example of ratios at work.



On a bicycle, the pedaling speed depends on the ratio of the front gear size compared to the rear gear. As the links of a chain go over the front gear, the very same number of links *must* be going over the rear gear.

If one gear is, say, twice as large as the other then one wheel is turning twice as fast as the other wheel.

The gear ratio is most easily computed by counting the number of teeth on each gear, and comparing them.

Example: What is the gear ratio of Mr. Hansen's mountain bike in the *lowest* gear? In the *highest* gear?

The "lowest" gear is the smallest front with largest rear gear. Counting teeth finds both sprockets have 28 teeth, which is a ratio of 28 : 28, which reduces to 1 : 1.

The "highest" gear is the largest front with the smallest rear gear. Counting teeth finds 48 front teeth and 12 rear teeth. This results in a ratio of 48 : 12, which reduces to 4 : 1.

Tying Them All Together

Ratios and fractions are interchangeable, and they can also be written as a decimal or a percent.

Percent	Decimal	Fraction	Ratio
6%	0.06	6/100	0.06 : 1 or 3 : 50

Identifying Equal Ratios

Since a ratio is just another fraction, multiplying or dividing both terms by the same number does not change the ratio.

Example: Kyle is the quarterback for his school football team. He completed 10 out of 15 passes in last Saturday's game. His friend Scott was the quarterback for the opposing team. Scott completed 12 passes out of 16. Compare the ratios of their pass completions to their pass attempts. Are they equal?

Solution 1: ***Equivalent Fractions***

One way to compare ratios is to rename both fractions with a common denominator, and then compare the numerators.

Kyle's completed pass ratio = 10 : 15 or $10/15$

Scott's completed pass ratio = 12 : 16 or $12/16$

$10/15$ reduces to $2/3$
$12/16$ reduces to $3/4$
} common denominator = 12

So re-write the fractions using the common denominator:

$2/3 = 8/12$
$3/4 = 9/12$
} $8/12 \neq 9/12$ *ratios are not equal*

Solution 2: ***Cross Product Test***

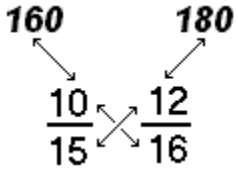
Another way to compare ratios or fractions is to use the cross product test: This is usually the easiest way to compare fractions at a glance.

Multiply the numerator of one fraction by the denominator of the other. Do this for both fractions, and write the answers. If the two products are equal, then the fractions are equal.

$\frac{10}{15} \nearrow\!\searrow \frac{12}{16}$

$10 \times 16 = 160$
$15 \times 12 = 180$
$160 \neq 180$ *ratios are not equal*

Here's another neat thing about the cross product test. It can easily tell you which fraction is larger! Just write the product *above* its numerator. Then compare the numbers in the same order you wrote them. For example:



the right hand ratio is larger

Solution 3: ***Decimal Comparison***

Another way to compare ratios is to grab your calculator, and compute the decimal number.

10/15 = 0.6666666666667
12/16 = 0.75 *ratios are not equal*

Dividing A Whole Quantity According To A Ratio

Example: Three hundred tents have to be divided between 2 scout troops at the ratio of 1 to 2. How many tents does each troop get?

Step 1: Add the terms of the ratio to get the total parts.
Ratio = 1 : 2 Adding terms = 1 + 2 = 3

Step 2: Determine fractional part of total each term represents.
Fractional parts = ⅓ and ⅔

Step 3: Multiply the total number of tents by fractional parts.

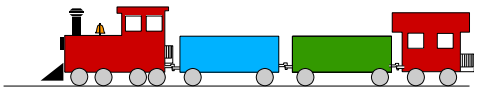
$\frac{1}{3} \times 300 = 100$ tents

$\frac{2}{3} \times 300 = 200$ tents

One troop gets 100 tents, and the other troop get 200.

Check: $100 : 200 = \frac{100}{200} = \frac{1}{2}$ or $1 : 2$

Example: A model train leaves city A at the same time another model train on the same track leaves city B. Train A is moving three times faster than Train B. City A and B are 200 inches apart. Obviously the trains will soon collide. How far is the point of collision from city A?



Solution: You recognize this is a "ratio" problem because the collision point will depend on the ratio of the train speeds. That is, the faster one train moves relative to the other, the more distance it will cover.

In this problem, the actual speed in mph is not important; the speed may dictate *when* the trains collide, but not *where*. If the speed ratio is constant, the distance covered by each train is constant.

Add the terms of the ratio to get the total parts.
Ratio = 3 : 1 Adding terms = 3 + 1 = 4

Determine fractional part of total each speed represents.
Fractional parts = ¾ and ¼

Multiply whole quantity (distance) by fractional parts.

$\frac{3}{4} \times 200$ inches $= 150$ inches

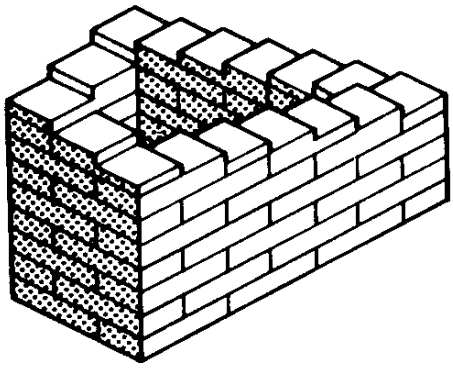
The model trains will collide 150 inches from city A.

Vocabulary

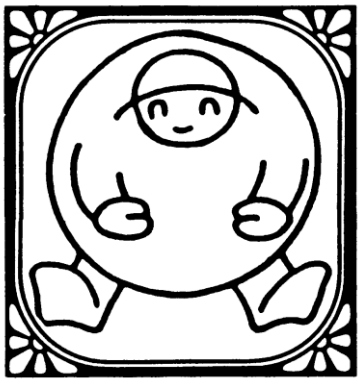
- *Ratio* - the relationship between two numbers or values. A ratio is a way of comparing two quantities. A ratio can be treated like a fraction.
- *Cross product test* - a way to compare two ratios to see if they are equal. Write the ratios as fractions, and form a new number by multiplying each numerator by the other fraction's denominator. If the numbers are equal, the ratios are equal.
- *Equivalent fraction test* - a way to compare two ratios to see if they are equal. Write the ratios as fractions, then adjust them to have a common denominator. Then you can simply compare the numerators.

Just for Fun…

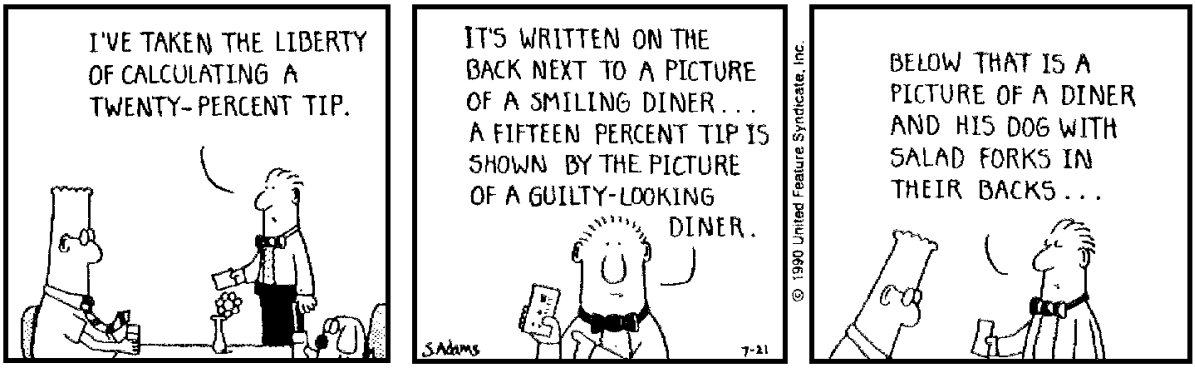
Point to the top step. Now point to the bottom step. Something is wrong…



This math club student has eaten too many donuts! Turn this upside down to see what happens if he keeps on eating.



Dilbert, by Scott Adams



1) Write each ratio as a fraction. Do not reduce.

a) Pennies to dimes _____

b) Dimes to quarters _____

c) Quarters to total coins _____

d) Heads to tails _____

e) Eyeballs to ear lobes _____

2) Use the cross product test to compare these fractions, and write *equal* or *not equal* for your answer. Check your work using a calculator or the equivalent fraction method.

a) $\frac{5}{6}$ $\frac{40}{54}$ *Example:* $5 \times 54 = 270,\ 40 \times 6 = 240,$ *not equal*,

Check: *0.8333, 0.7407*

b) $\frac{5}{6}$ $\frac{9}{21}$

c) $\frac{18}{27}$ $\frac{5}{45}$

d) $\frac{4}{17}$ $\frac{16}{54}$

e) $\frac{3}{7}$ $\frac{15}{35}$

f) $\frac{12}{15}$ $\frac{48}{60}$

3) Problem: 80 math books are to be divided between the fourth and fifth grade classes at Sunny Hills at the ratio of 3 : 5. The fifth grade class will get the most books.

a) How many books will each class receive?

b) What is the ratio of fifth grade books to the total number of books?

c) What percent of the total books will the fourth grade receive?

4) A professional basketball season is 82 games long. Sam's favorite team won 60 games and lost 22 games.

a) What is the ratio of wins to losses?

b) What is the ratio of wins to games played?

5) Sunny Hills math club presently has 25 fourth graders, and 20 fifth graders.

a) What is the ratio of fourth graders to fifth graders?

b) What is the ratio of fourth graders to total students?

6) Suppose a family has four (normal!) people.

a) What is the ratio of noses to toes?

b) What is the ratio of fingers to ears?

c) Would these ratios change if there were more people in the family?

7) Who says you can't compare apples and oranges?! Suppose a grocery sack contains six apples and twelve oranges.

a) What is the ratio of apples to oranges?

b) What is the ratio of apples to fruit?

8) Mental Math. Do these in your head, and then *check your answer* with pencil and paper, or calculator, or parent, or whatever.

a) Round the number 29.8 to the nearest whole number, then multiply by 3.

b) Circle *yes* or *no*: Is 981 evenly divisible by 3?

c) What is the ratio of *all* even numbers to *all* odd numbers?

d) Last week, what was your name?

e) Circle *true* or *false*: The LCM of any two prime numbers is the product of those two numbers.

f) Circle *up* or *down*. To round 0.5 to the nearest unit, do you round up to 1 or down to zero?

g) Rearrange the digits in the number 603 so that when you divide it by 3, the result is 120.

h) Suppose a music CD costs $8. How many quarters would you need to buy it?

i) Circle *yes* or *no*: Before you can <u>multiply</u> two fractions, must you always change them to a common denominator?

j) Round off both 97 and 11 to the nearest 10, then multiply the two rounded numbers. What is the product?

You're done! Detach the homework from the lesson, and turn in just the *homework*.

