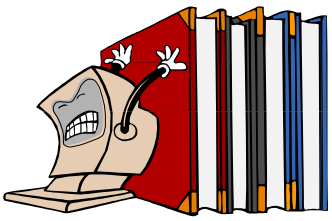

Like fractions and decimals, **percents** are a way of representing part of something.

Why do we use percentages? It is a simple and convenient way to use *ratios*. It is easier to say "five percent" than "zero point zero five of the total".

You can use percents anywhere fractions are used, and that's just about everywhere! Here are some examples…

Percents can compare groups: "80% of the class likes chocolate doughnuts".
Percents can predict the future: "in basketball he makes 40% of his free-throws".
Percents can simplify many numbers: "she got 99% on her math tests this year".

Percents are particularly useful with money (did you ever hear of sales tax?) because both dollars and percents have units of hundredths.

Percent

Percent or % is a special ratio that means a number of parts "per one hundred". It is a standard of measure.

Examples:
10% may be written as $\frac{10}{100}$
25% may be written as $\frac{25}{100}$
99% may be written as $\frac{99}{100}$
125% may be written as $\frac{125}{100}$

Definitions:
Base - is the whole quantity
Percentage - is the portion of the whole quantity
Rate - is the percent (%), think of "ratio" when you see this!

$$\frac{\text{Percentage}}{\text{Base}} = \%\text{Rate}$$

Percents may be added, subtracted, multiplied or divided in the same way as other ratios.

Example: 8% + 6% = 14%

Example: 18% - 12% = 6%

Example: 18% ÷ 9 = 2%

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Converting Percents To Decimals

To change a percent to a decimal, remove the percent sign and move the decimal point two places to the left (divide by 100).

Example: Change 25% to a decimal

$$25\% = \frac{25}{100} = \frac{1}{4} = 0.25$$

% ⇨ n.nn

Example: Change 200% to a decimal

$$200\% = \frac{200}{100} = 2$$

Converting Decimals To Percents

To change a decimal to a percent, multiply by 100 (move the decimal point **two places** to the right) and write the percent sign:

Example: Change 0.057 to a percent

$$0.057 = 5.7\%$$

n.nn ⇨ %

Example: Change 2.43 to a percent

$$2.43 = 243\%$$

Computing Percentages And Percents

Example: What is 8% of 50 ?

Solution: The rate or *percent* is 8%. The *base* or whole quantity is 50.
Remember that a "*percent*" is the same as a ratio.
Also recall that "*of*" is the same as multiplying.
First, convert the percent to a decimal:

$$8\% = 0.08$$

Next, multiply the decimal by the base:

$$50 \times 0.08 = 4$$

Answer: 8% of 50 is 4

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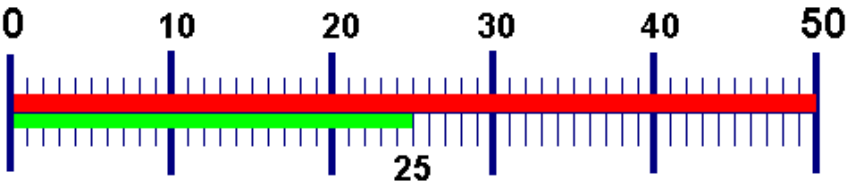
Example: 25 is *what percent* of 50?



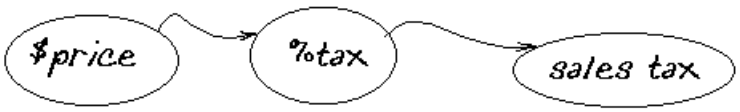
Solution: To solve a "*what percent"* problem, we must find the *ratio*.
Here the *base* or whole quantity is again 50.
The percent or part of the whole quantity is 25.
To solve for the percent, divide the percent 25 by the base 50.

$$25 \div 50 = \frac{25}{50}$$
$$= \frac{1}{2}$$
$$= 0.5$$

Now convert the decimal fraction 0.5 into percent by multiplying by 100 and writing %:

$$= 50\%$$

Finding Sales Tax



To compute the dollar amount of sales tax, given a price and a percent tax:

1. Rename the percent as a decimal.

2. Multiply the price by the decimal tax rate.

Example: Dylan bought his father a $22 shirt for Christmas. The sales tax on clothing is 8.6%.
How much tax did Dylan pay? What was the total cost of the shirt?

Solution: To find the amount of tax, we need to find 8.6% of $22.

Rename the percent as a decimal: 8.6% = 0.086

Multiply the cost by the tax rate: $22 × 0.086 = $1.89

Find the total cost by adding tax: $22.00 + $1.89 = $23.89

On some calculators, you can compute the sales tax directly. Some calculators have a 'percent' key to help do this.

Example: What is 8.6% of $35?

Solution: Press 8 . 6 2nd % and it displays 0.086
Press × 35 = and it displays 3.01, which means $3.01

Common Mistakes With Percents

Since percent is a ratio, *read carefully* to see what it is a ratio of ! What is the *base* of the percentage? For example, compare these:

a) Hannah got 80% as much Halloween candy as Jared.

b) Hannah got 80% more Halloween candy than Jared.

So if Jared got two pounds of candy, how much candy did Hannah get? The two statements above are a lot different!
How do you change the words "as much" and "more" into arithmetic?

1. *"as much as"* ⇒ Hannah got 2 pounds *times* 0.8, or 1.6 pounds of candy.

2. *"more than"* ⇒ Hannah got 2 pounds *plus* 2 *times* 0.8, or 3.6 pounds of candy. (Wow!)

Another common mistake is with combining *percents over time*. The *base* or beginning of one time period is the same as the *end* of the last time period. Since percents are a ratio, and ratios mean multiplication, it is *not* correct to *add* percents to changes over time.

Example: Which option do you prefer? Or are they the same?

a) Keep your current allowance steady?

b) Raise your allowance 20% this month, lower it 20% next month, then keep it steady?

Solution: At first glance it seems the same. Adding and subtracting 20% gives you a bonus and then cancels out, right? *Wrong!*

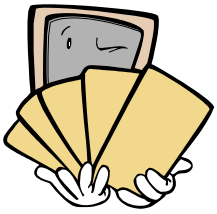
You can't add or subtract percents that have different bases or happen at different times!
Let's assume you get $10 monthly allowance, and work it out…

a) This month and every month you get $10.

b) This month you get $10 plus 20% of $10, which is $12.
Next month you get $12 minus 20% of $12, which is $9.60
Now your allowance is steady but at a lower level!

Answer: (a) 100% of Math Club students will prefer keeping their allowance steady.

Exponents and Powers of Ten



To write one million as a numeral takes a 1 and *six* 0's: 1,000,000.
To write one billion, you use a 1 and *nine* 0's: 1,000,000,000.
One trillion is even larger and uses three more 0's: 1,000,000,000,000.

Large numbers have always been interesting to mathematicians. Writing lots of zeros is *not* so interesting. There is a way to write abbreviations for large numbers, without all those pesky zeros!

- One million (1,000,000) can be written 10^6.
- One billion (1,000,000,000) can be written 10^9.
- One trillion (1,000,000,000,000) can be written 10^{12}.

The little number is the same as the number of zeros, and is called an *exponent*. An exponent tells how many times to multiply the other number, called the *base*, by itself. So 10^6 equals $10 \times 10 \times 10 \times 10 \times 10 \times 10$, and that equals 1,000,000. (You can check this on your calculator.)

To read a power of ten, such as 1,000, you can say "one thousand" or "ten to the third power." One million is "ten to the sixth power."

With numbers that aren't exactly a power of ten, 13 million (13,000,000) for example, mathematicians use shorthand that still uses exponents with ten as the base. Since 13 million is 13 times 1 million, you can write it as 13×10^6. That's a standard abbreviation called *scientific notation*. . This is displayed on some handheld calculators as "13 E6".

Exponents are especially useful when the powers of ten get even bigger. Here is a chart that shows how handy mathematical shorthand can be.

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BIG **Power of Ten**	**Name**	
10^{0}	One	
10^{3}	Thousand	kilo
10^{6}	Million	mega
10^{9}	Billion	giga
10^{12}	Trillion	tera
10^{15}	Quadrillion	exa
10^{18}	Quintillion	
10^{21}	Sextillion	
10^{24}	Septillion	
10^{27}	Octillion	
10^{30}	Nonillion	
10^{33}	Decillion	
10^{36}	Undecillion	
10^{39}	Duodecillion	
10^{42}	Tredecillion	
10^{45}	Quattuordecillion	
10^{48}	Quindecillion	
10^{51}	Sexdecillion	
10^{54}	Septendecillion	
10^{57}	Octodecillion	
10^{60}	Novemdecillion	
10^{63}	Vigintillion	
10^{100}	Googol	
10^{googol}	Googolplex	

teeny weeny **Power of Ten**	**Name**	
10^{0}	One	
$10^{-3} = 1/10^{3}$	Thousandth	milli
$10^{-6} = 1/10^{6}$	Millionth	micro
$10^{-9} = 1/10^{9}$	Billionth	nano
$10^{-12} = 1/10^{12}$	Trillionth	pico
$10^{-15} = 1/10^{15}$	Quadrillionth	femto

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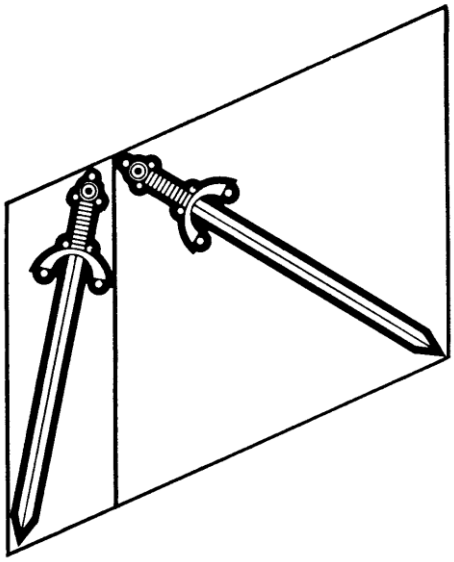
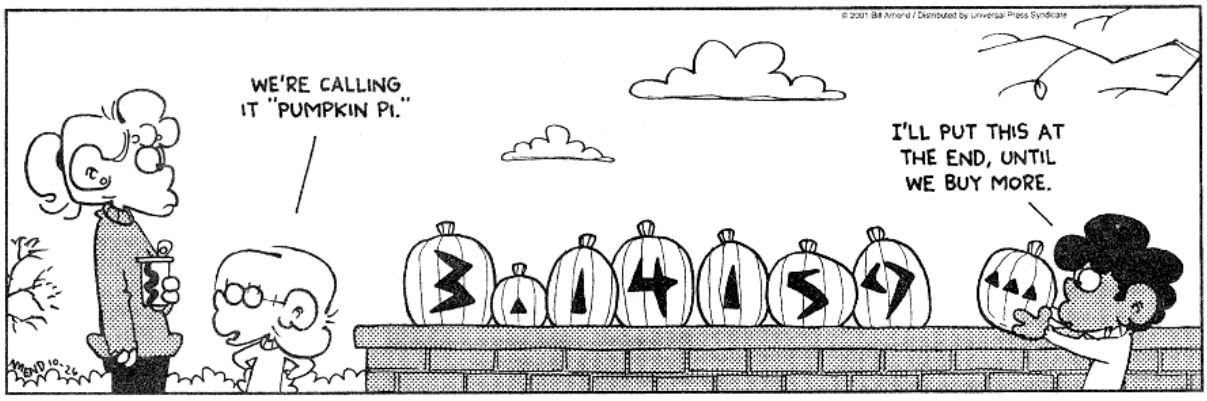
Just for Fun…

What do you see in the middle of the frame? Is it a letter "B" or the number "13"?

These two swords are drawn in a parallelogram. Which looks bigger?



Foxtrot, by Bill Amend



Vocabulary

- *"As much"* – always means multiply.
  For example, "She got 120% as much as he did" means to multiply his amount by 120% or 1.2.

- *"More than"* – always means add.
  For example, "She got 20% more than he did" means to take 100% of his amount and add 20% more. (Notice this is equivalent to multiplying by 120% because 100%+20%= 120%)

- *Percent, Percentage, %* - a number of parts per 100. It is a standard of measure and comparison.

- *Base* - represents (or is) the whole quantity

- *Rate* - percent (%), think of "ratio" when you see this!

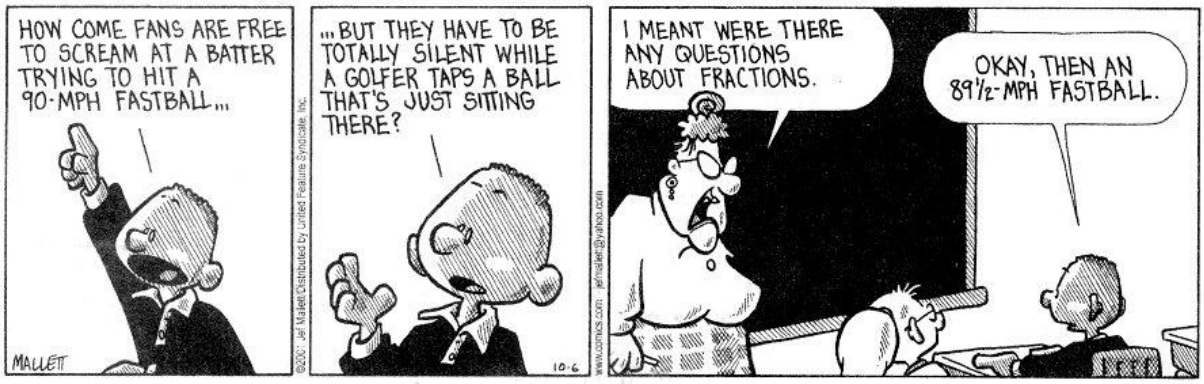
- *Interest* - money paid for the use of borrowed money. Interest rate is a percentage per unit of time.

- *Sales tax* - money collected at the point of sale and is computed on a percentage (8.6% in Issaquah) of the listed price. This is why a $1.95 toy really costs you $2.12.

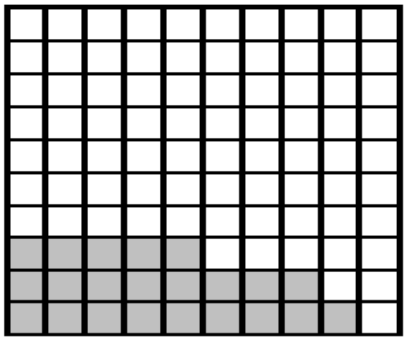
- *Scientific notation* - a method of writing numbers as a decimal number times a power of ten. For example, 3 trillion is written as 3×10^{12}. This is displayed on some handheld calculators as "3.0 E12".

- *Exponent* - an exponent tells how many times to multiply the other number, called the *base*, by itself. For example, "ten to the exponent three" is 10^3 or 1,000.

Frazz – by Jef Mallett



1) Write the percent:



a) of the shaded squares

b) of the unshaded squares

2) Convert these decimal numbers to a percent:

a) 0.39 *Example: 39%*

b) 0.05

c) 0.077

d) 0.9

e) 0.999

f) 1

g) 1.45

h) 2.5

3) Convert these percents to a decimal number:

a) 40% *Example: 0.4*

b) 20%

c) 30%

d) 7.5%

e) 22.5%

f) 99%

g) 107%

h) 200%

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4) Multiply these percents:

a) 4% of 50 = *Example: 0.04 × 50 = 2*

b) 16% of 100 =

c) 16% of 200 =

d) 20% of 150 =

e) 100% of 140 =

f) 10% of 1000 =

g) 25% of 25 =

5) Compute the resulting percent for these ratios:

a) 25 is what percent of 75 ? *Example:* $\frac{25}{75} = 0.33\overline{3} = 33\%$

b) 1 is what percent of 4 ?

c) 2 is what percent of 8?

d) 50 is what percent of 200 ?

e) 100 is what percent of 200 ?

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6) Write these percents as a fraction (always reduce fractions!):

a) 6% *Example:* $\frac{6}{100} = \frac{3}{50}$

b) 90%

c) 40%

d) 18%

e) 12.5%

f) 37.5%

7) Solve these problems:

a) In a telephone poll 56 out of every 100 people had a home computer. What percent of people had home computers?

b) Cameron bought a Pentium-powered Game Boy® for $245.35. The sales tax on the purchase (Intel Inside!) was charged at a rate of 8%. How much money did Cameron pay in tax?

c) Math club members wanted to raise $200 to give generously to their wonderful Math Club coach. (He's good-looking, too.) So far they have raised $125. What percent of their goal do they have left to raise?

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d) Kevin scored 88.8% on a test. Chris received 8 out of 9 correct. Write these two scores as a percentage. Who has the higher score?

e) Mr. Hansen took his family to dinner at Red Robin. The cost of food was $50.00. If sales tax was 8.6% and he left a tip that was 15% of the food, how much did he spend all together?

f) Esther is trying to earn $75 to buy a special calculator for Math Club. (She just can't get enough of these problems!) She convinces a local bike shop owner to give her a 15% commission on new bikes she sells outside of the store. If each bicycle sells for $200, how many will she need to sell?

8) You don't have to use exponents just for powers of ten. You can have powers of two, three, or of any number. The rule is still the same: The exponent tells how many times to multiply the base by itself. Answer the following:

a) $2^3 =$ *Example:* $2 \times 2 \times 2 = 8$

b) $3^2 =$

c) $2^6 =$

d) $4^3 =$

e) Is $\frac{2^4}{2^3}$ the same as 2^{4-3}? Circle *yes* or *no*.

f) Is $2^4 \times 2^5$ the same as 2^{4+5}? Circle *yes* or *no*.

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9) Mental Math: do these in your head, and write down the answers. When you're all done, *check your answer* with pencil and paper, or with a calculator

a) What is $20/36$ after it is reduced?

b) What is the least common multiple of 6 and 10?

c) What is $\frac{3}{10}$ divided by $\frac{2}{10}$?

d) What is your name?

e) To be evenly divisible by 3, the sum of the digits in a number must be divisible by 3. Circle ***yes*** or ***no***: Is 8328 evenly divisible by 3?

f) To be evenly divisible by 9, the sum of the digits in a number must be divisible by 9. Circle ***yes*** or ***no***. Is 8328 evenly divisible by 9?

g) In an *improper* fraction, the numerator is larger than the denominator. Circle *TRUE* or *FALSE*?

You're done! Detach the homework from the lesson, and turn in just the *homework*.

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