SHS Math Club
2009-10
Math Toolbox

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I. **Sequences**

a. Arithmetic Sequence
   i. Definition= in an arithmetic series, the difference between terms is constant. Ex: 4 + 10 + 16 + 22… they all have a difference of six between terms.
   ii. Formula for sum of terms in sequence= (1/2) * n * (a+z)
      1. n= number of terms in the sequence
      2. a = first term in sequence
      3. z = last term in sequence
   iii. Answer to example if the sequence goes up to 100 is : (1/2) x 17(4+100) = 884

b. Geometric Sequence
   i. Definition= it is a sequence such that each successive term is obtained from the previous term by multiplying a fixed number called a ratio. Ex: 3 + 6 + 12 + 24 + 48 + 96
   ii. Formula for sum of terms in sequence
      1. *Finite Geometric Sequence* = $(a(1-r^n))/(1-r)$
         a. a= first term in sequence
         b. r= common ratio
         c. n= number of terms
         d. note: r does not equal 0
      2. *Infinite Geometric Sequence* = a/(1-r)
         a. If the absolute value of r is greater than or equal to one, then the infinite series does not have a sum

c. Figurative Number Sequences
   i. Triangular numbers
      1. Equation = n(n+1)/2.
      2. First terms of sequence= 1, 3, 6, 10, 15, 21, 28
      3. n= order of number in sequence. Ex: if n = 3, then look for the third term in the sequence (3(4)/2) = 6.
   ii. Square numbers
      1. Equation = n^2
      2. First terms of sequence = 1, 4, 9, 16, 25, 36, 49
      3. n= order of number in sequence.
      4. Try this: What is the sum of the first "n" odd positive numbers??
   iii. *Pentagonal numbers + beyond* = n(3n-1)/2, n(4n-2)/2, n(5n-3)/2…

II. **Geometric Formulas**

a. The Pythagorean Theorem
   i. $a^2 + b^2 = c^2$
   ii. only works when "a" and "b" are the lengths of the legs of a right triangle, and "c" is the hypotenuse
   iii. Pythagorean triples:
      1. 3, 4, 5 (3^2 + 4^2 = 5^2) (9 + 16 = 25)
      2. 8, 15, 17 (8^2 + 15^2 = 17^2) (64 + 225 = 289)
      3. 5, 12, 13
      4. 7, 24, 25
      5. 20, 21, 29
      6. 12, 35, 37
      7. 9, 40, 41
      8. 11, 60, 61
      9. Note: If you double "a" and "b", and "c", you can get a Pythagorean triple that is also the same as the old one (ex: 3, 4, 5 double = 6, 8, 10 (36 + 64 = 100)). Those triples are not repeat listed here.

b. Triangle Formulas (not including Pythagorean)
   i. The 45-45-90 degree triangle
      1. Properties are:
         a. It is half a square
         b. Hypotenuse = S * sqrt(2) if S is the side length of the leg
   ii. The 30-60-90 degree triangle
      1. Properties are:
         a. Half of an equilateral triangle
         b. If shortest leg = S, then hypotenuse = 2S, and the long leg = S squareroot(3)
   iii. Sine, Cosine, and Tangent
      1. Sine of an angle = opposite leg / hypotenuse
      2. Cosine of an angle = adjacent leg / hypotenuse
      3. Tangent of an angle = opposite leg / adjacent leg
      4. $\sin^2 x + \cos^2 x = 1$
      5. tan(x) = sin(x) / cos(x)
      6. sin(x+y) = sin(x)cos(y) + cos(x)sin(y)
      7. cos(x+y) = cos(x)cos(y) + sin(x)sin(y)
      8. tan(x+y) = (tan(x) + tan(y))/(1+tan(x)tan(y))
      9. Note: you are not expected to memorize 6, 7, 8 they are just for fun, and 1, 2, 3 ONLY work in a right triangle!!!
      10. Many… many… more
   iv. Heron's Formula
      1. Area of triangle = sqrt((sp) * (sp - a) * (sp - b) * (sp - c))
         a. Sp= semi-perimeter
         b. a= first side, b= second side, c = third side
         c. Works for any triangle

c. Properties of Polygons
    i. Number of diagonals in a polygon with "n" sides
        1. n(n-3)/2
    ii. Number of degrees in an n-sided polygon
        1. 180(n-2)
    iii. Sum of the exterior angles of any polygon
        1. 360 degrees

III. **Miscellaneous Geometry**

a. Space Diagonal of a cube : S * sqrt(3) where S is one length of the cube
b. Areas and Volumes
    i. Area of Square = s^2
    ii. Area of Square = $(d^2)/2$
        1. d = diagonal
    iii. Area of a rhombus $(d_1 * d_2)/2$
        1. d1 and d2 are the diagonals
    iv. Area of triangle = ½ * b*h
        1. b=base, h=height
    v. Area of triangle = ½ * b * c * sin(A)
        1. b= adjacent side 1, c = adjacent side 2, and A = the angle for which the sides b,c are adjacent to
    vi. Area of circle = (pi) * r^2
        1. r= radius
    vii. Area of trapezoid = ½ * $h(b_1 + b_2)$
        1. h = height, b_1;b_2 are the two bases
    viii. Volume of cylinder= B * h
        1. B= area of base and h= height
    ix. Volume of cone= (1/3) * B * h
    x. Volume of sphere = (4/3) * (pi) * r^3
    xi. Surface area of sphere = 4 * (pi) * r^2

IV. **Factoring Method**

a. Factor out any common factors
b. If it is two terms, check whether it can be rewritten as the difference of two cubes, or the sum of two cubes
c. If it is three terms, check for the perfect square trinomials
d. If it has more than three terms, try to factor by grouping

V. **Combinations/Permutations**

a. Number of permutations of "n" different things:
   i. n!
b. Number of permutations of "n" things where "r" things are the same:
   i. n!/r!
c. Number of permutations of "n" things taken "r" at a time:
   i. n!/(n-r)!
d. Number of combinations of "n" things taken "r" at a time:
   i. n!/(r! * (n-r)!)
   ii.

VI. **List of basic squares/cubes**

Number (n)	Square (n^2)	Cube (n^3)
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728

VII. **List of Factorials**

a. Expressed as "n!"
b. n! = n * (n-1) * (n-2) * (n-3)… * 3 * 2 * 1

Number (n)	Factorial (n!)
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800

